Hidden Markov Model

Da Huo

08/15/2018

This is the Hidden Markov Model we constructed for predicting the probability of The Coffee Pot is empty. In this model, the state X is whether the pot is empty. $X = \{E, \neg E\}$ The observation r here is how many times the pot is been removed. $R = \{1, 2, ..., n\}$ where n is the last time coffee pot been removed in one coffee making cycle. When we detect the coffee machine is on again, we reset r and count from 1 again.

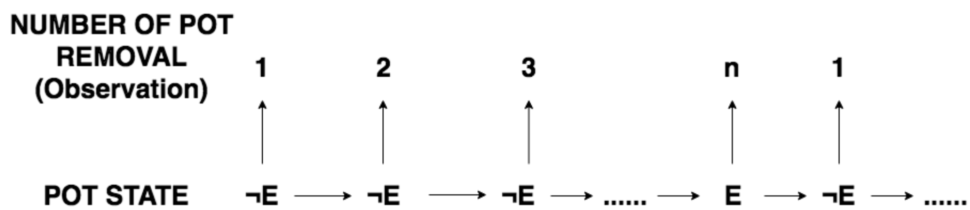

Figure 1: HMM of Coffee Predicting

Priors in Inference

Transition Model

$$P(X_{t+1} = E | X_t = E) = 0 \tag{1}$$

Here, we assume the when pot is empty, the next time we receive a data point is when making coffee is detected and coffee pot is definitely not empty.

$$P(X_{t+1} = E | X_t = \neg E) = \frac{1}{\text{\# of pot removals within one cycle of making coffee}} \tag{2}$$

Sensor Model

$$P(R_t = n|X_t = E) = \frac{\text{\# of } R \geq n|E}{\text{\# of E}} \tag{3}$$

$$P(R_t = n|X_t = \neg E) = \frac{\text{\# of } R \geq n|\neg E}{\text{\# of } \neg E} \tag{4}$$

Inference

Transition Matrix

$$T = \begin{bmatrix} P(X_{t+1} = E|X_t = E) & P(X_{t+1} = \neg E|X_t = E) \\ P(X_{t+1} = E|X_t = \neg E) & P(X_{t+1} = \neg E|X_t = \neg E) \end{bmatrix} \tag{5}$$

Observation Matrix

$$O_{R=n} = \begin{bmatrix} P(R_t = n|X_t = E) & 0 \\ 0 & P(R_t = n|X_t = \neg E) \end{bmatrix} \tag{6}$$

Inference

Let $f_i = P(X_i)$,

$$f_{t+1} = \alpha f_t T O_{R_{t+1}=n} \tag{7}$$